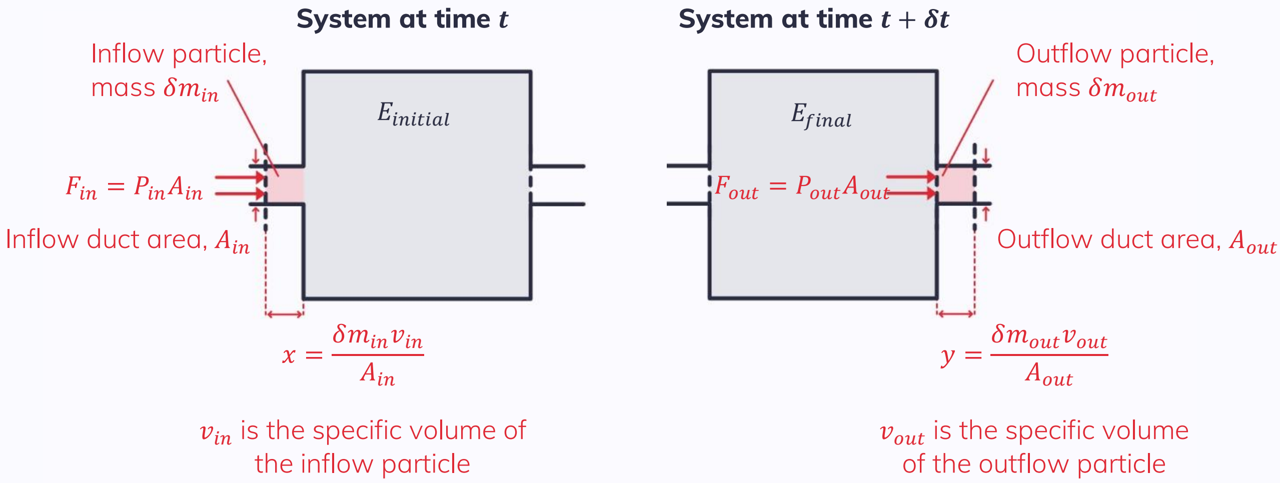
To derive the Steady Flow Energy Equation, we can model a control volume as a system that changes shape:



Everything that is shaded grey is the closed system, and the red part is a minute particle entering/exiting the closed system.

Solving for Work Done

The system produces an output shaft work:

$$\delta W_{shaft}$$

Work done by system on its surroundings:

$$W_2 = F_2\, y$$

$$W_2 = P_{out}\, A_2 \left(\frac{\delta m_{out}\, v_{out}}{A_2}\right)$$

$$W_2 = \delta m_{out}\, P_{out}\, v_{out}$$

Work done by surroundings onto system:

$$W_1 = F_1 x$$

$$W_1 = P_{in}\, A_1 \left(\frac{\delta m_{in}\, v_{in}}{A_2}\right)$$

$$W_1 = \delta m_{in}\, P_{in}\, v_{in}$$

Total Work done:

$$W = \delta W_{shaft} + W_2 - W_1$$

$$\delta W = \delta W_{shaft} + \delta m_{out}\, P_{out}\, v_{out} - \delta m_{in}\, P_{in}\, v_{in}$$

Solving for Energy

The fluid particle entering the system has mass δm_{in} volume v and is travelling at speed C_{in}. It has total energy made up of its internal energy, U_{in}, kinetic energy, $\frac{1}{2}\delta m_{in} C_{in}^2$, and potential energy $\delta m_{in} gZ$ (were Z is its vertical position relative to a given zero point):

$$Energy\ of\ fluid\ particle\ entering = U_{in} + \frac{1}{2}\delta m_{in} C_{in}^2 + \delta m_{in} gZ$$

$$= \delta m_{in}\left(u + \frac{1}{2}C^2 + gZ\right)_{in}$$

Similarly, the fluid particle leaving the system has mass δm_{out} volume v and is travelling at speed C_{out}. It has total energy made up of its internal energy, U_{out}, kinetic energy, $\frac{1}{2}\delta m_{out} C_{out}^2$, and potential energy $\delta m_{out} gZ$:

$$Energy\ of\ fluid\ particle\ leaving = U_{out} + \frac{1}{2}\delta m_{out} C_{out}^2 + \delta m_{out} gZ$$

$$= \delta m_{out}\left(u + \frac{1}{2}C^2 + gZ\right)_{out}$$

Since the energy initially in the system is $E_{initial}$ and that at time $t + \delta t$ is E_{final}, the total change in energy is the difference between these + the energy brought in by the fluid particle – the energy of the leaving particle:

$$E_2 - E_1 = E_{final} - E_{initial} + \delta m_{out}\left(u + \frac{1}{2}C^2 + gZ\right)_{out} - \delta m_{in}\left(u + \frac{1}{2}C^2 + gZ\right)_{in}$$

Applying the first law

Now that we know the total change in energy and work, we can use the **first law of thermodynamics:**

$$\delta Q - \delta W = E_2 - E_1$$

$$\delta Q - (\delta W_{sh} + (\delta m P v)_{out} - (\delta m P v)_{in}) = E_f - E_i + \delta m_{out}\left(u + \frac{1}{2}C^2 + gZ\right)_{out} - \delta m_{in}\left(u + \frac{1}{2}C^2 + gZ\right)_{in}$$

$$\delta Q - \delta W_{sh} = E_f - E_i + \left[\delta m\left(\underbrace{u + Pv}_{u + Pv = h} + \frac{1}{2}C^2 + gZ\right)\right]_{out} - \left[\delta m\left(\underbrace{u + Pv}_{u + Pv = h} + \frac{1}{2}C^2 + gZ\right)\right]_{in}$$

$$\delta Q - \delta W_{sh} = E_f - E_i + \left[\delta m\left(h + \frac{1}{2}C^2 + gZ\right)\right]_{out} - \left[\delta m\left(h + \frac{1}{2}C^2 + gZ\right)\right]_{in}$$

Differentiating with respect to time:

$$\underbrace{\frac{dQ}{dt}}_{\dot{Q}} - \underbrace{\frac{dW_{sh}}{dt}}_{\dot{W}_{sh}} = \frac{d}{dt}(E_f - E_i) + \left[\underbrace{\frac{dm}{dt}}_{\dot{m}}\left(u + Pv + \frac{1}{2}C^2 + gZ\right)\right]_{out} - \left[\underbrace{\frac{dm}{dt}}_{\dot{m}}\left(u + Pv + \frac{1}{2}C^2 + gZ\right)\right]_{in}$$

For steady flow, $E_f = E_i$ so this term becomes zero

Therefore, this can be rewritten as:

$$\dot{Q} - \dot{W}_{sh} = \left[\dot{m}\left(h + \frac{1}{2}C^2 + gZ\right)\right]_{outflow} - \left[\dot{m}\left(h + \frac{1}{2}C_{gZ}^2\right)\right]_{inflow}$$

Note: The two parts on the right-hand side represent the **total outflow and total inflow.** Therefore, we need to accommodate for **multiple ports:**

$$\dot{Q} - \dot{W}_{sh} = \sum_{outflow}\left[\dot{m}\left(h + \frac{1}{2}C^2 + gZ\right)\right] - \sum_{inflow}\left[\dot{m}\left(h + \frac{1}{2}C^2 + gZ\right)\right]$$

And there you have it. The Steady Flow Energy Equation.

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