Engineering Notes.net

Structures Key Equations

Statical Determinacy

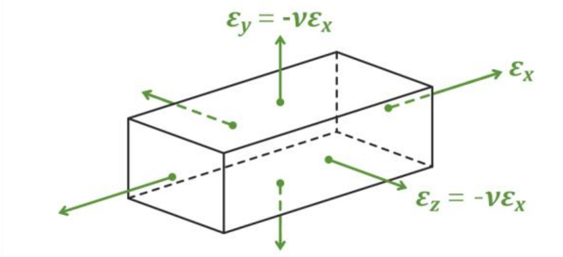
Mechanism	$b + n < 2j$
May be determinate	$b + n = 2j$
Indeterminate	$b + n > 2j$

- b is the number of bars
- n is the number of reactions
- j the number of pins

Stress & Strain

Engineering Stress	$\sigma = \frac{F}{A}$
Engineering Strain	$\varepsilon = \frac{x}{L}$
Youngs Modulus	$E = \frac{\sigma}{\varepsilon}$
Poisson's Ratio	$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x$
Hydrostatic Stress	$\sigma_H = \sigma_x = \sigma_y = \sigma_z$

Rectangular Cross-Sections



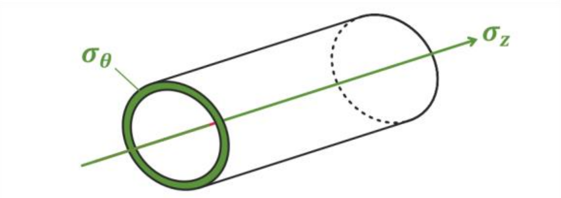
Change in Volume	$\Delta V = \varepsilon_V \times V$
Volumetric Strain	$\varepsilon_V = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$
Bulk Modulus	$K = \frac{\sigma_H}{\varepsilon_V}$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right) + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) + \alpha\Delta T$$

Cylindrical Thin-Walled Pressure Vessels



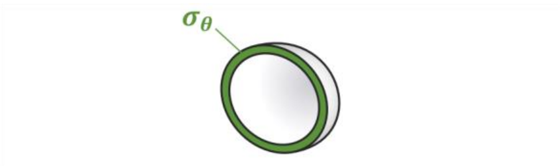
Hoop Stress, σ_θ	$\sigma_\theta = \frac{PR_i}{t}$
Axial Stress, σ_z	$\sigma_z = \frac{\sigma_\theta}{2} = \frac{PR_i}{2t}$
Axial strain, ε_z	$\varepsilon_z = \frac{\Delta L}{L}$
Hoop strain, ε_θ	$\varepsilon_\theta = \frac{\Delta r}{r}$
Volumetric Strain	$\varepsilon_V = \frac{\Delta V}{V} = 2\varepsilon_\theta + \varepsilon_z$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_\theta + \sigma_r)\right) + \alpha\Delta T$$

$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu(\sigma_r + \sigma_z)\right) + \alpha\Delta T$$

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - \nu(\sigma_\theta + \sigma_z)\right) + \alpha\Delta T$$

Spherical Thin-Walled Pressure Vessels



Hoop Stresses, $\sigma_\theta, \sigma_\phi$	$\sigma_\theta = \sigma_\phi = \frac{PR_i}{2t}$
Hoop Strains, $\varepsilon_\theta, \sigma_\phi$	$\varepsilon_\theta = \varepsilon_\phi = \frac{\Delta r}{r}$
Volumetric Strain	$\varepsilon_V = \frac{\Delta V}{V} = 3\varepsilon_\theta$

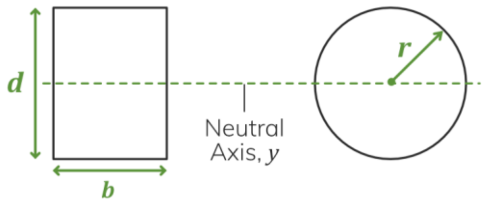
$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\left(\sigma_r + \sigma_\phi\right)\right) + \alpha\Delta T$$

$$\varepsilon_\phi = \frac{1}{E}\left(\sigma_\phi - \nu\left(\sigma_r + \sigma_\theta\right)\right) + \alpha\Delta T$$

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - \nu\left(\sigma_\theta + \sigma_\phi\right)\right) + \alpha\Delta T$$

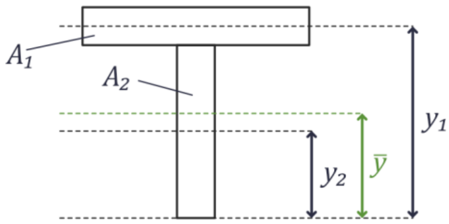
- Take radial stresses and strains as zero in both spheres and cylinders.
- Axial strain in a sphere is also zero.

Second Moment of Area



$$I = \frac{bd^3}{12}$$

$$I = \frac{\pi r^4}{4}$$



$$\bar{y}(A_{total}) = y_1(A_1) + y_2(A_2)$$

$$I_{xx} = [I_1 + A_1(y_1 - \bar{y})^2] + [I_2 + A_2(\bar{y} - y_2)^2]$$

Beam Theory

Key Relation $$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

Radius & Moment $$R = \frac{d^2v}{dx^2} = \frac{1}{EI}M$$

Slope Angle $$\frac{dv}{dx} = \frac{1}{EI}\int M\,dx$$

Deflection $$v = \frac{1}{EI}\iint M\ dx$$

Torsion in Thin-Walled Shafts

Key Relation $$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L}$$

Shear Stress & Strain $$\gamma = \frac{\tau}{G}$$

Shear Modulus $$G = \frac{E}{2(1+\nu)}$$

Torsional Stiffness, K_T $$K_T = \frac{T}{\theta} = \frac{JG}{L}$$

$$\frac{1}{K_{T,total}} = \frac{1}{K_{T1}} + \frac{1}{K_{T2}} \dots$$

Torsion in Thin-Walled Shafts

Shear Strain $$\gamma = \frac{GR_0}{L}$$

Shear Stress $$\tau = \frac{GR_0\theta}{L}$$

$$\tau = \frac{T}{2\pi R_0^2 t}$$

Torsion in Solid & Hollow Shafts

Shear Strain $$\gamma(r) = \frac{r\theta}{L}$$

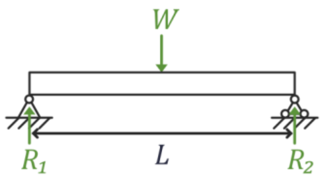
Shear Stress $$\tau = \frac{Gr\theta}{L}$$

Torque $$T = \frac{G\theta J}{L}$$

2nd Polar Moment of Area $$J = \frac{\pi D^4}{32}$$

Standard Slope & Deflection of Beams

Simply Supported with Point Mass



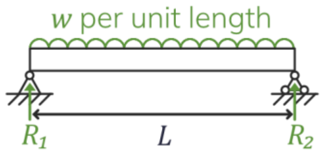
End Slope

$$\frac{WL^2}{16EI}$$

Central Deflection

$$\frac{WL^3}{48EI}$$

Simply Supported with Distributed Load



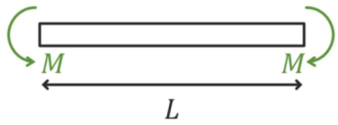
End Slope

$$\frac{wL^3}{24EI}$$

Central Deflection

$$\frac{5wL^4}{384EI}$$

Moment Supported



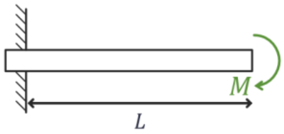
End Slope

$$\frac{ML}{2EI}$$

Central Deflection

$$\frac{ML^2}{8EI}$$

Cantilever with Moment



End Slope

$$\frac{ML}{EI}$$

End Deflection

$$\frac{ML^2}{2EI}$$

Cantilever with Point Load

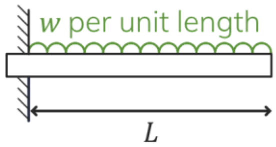
End Slope

$$\frac{WL^2}{2EI}$$

End Deflection

$$\frac{WL^3}{3EI}$$

Cantilever with Distributed Load



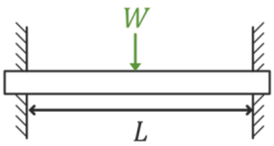
End Slope

$$\frac{wL^3}{6EI}$$

End Deflection

$$\frac{wL^4}{8EI}$$

Built in at Both Ends with Central Point Load



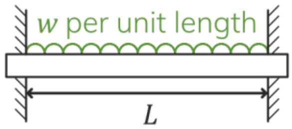
End Moment

$$\frac{WL}{8}$$

Central Deflection

$$\frac{WL^3}{192EI}$$

Built in at Both Ends with Distributed Load



End Moment

$$\frac{wL^2}{12}$$

Central Deflection

$$\frac{wL^4}{384EI}$$