

Thermodynamics Key Equations

Universal Gas Law

$$PV = n\bar{R}T = mRT \quad Pv = RT$$

Work Done

Always true

$$W_{12} = \int_{V_1}^{V_2} P dV$$

Constant P

$$W_{12} = P(V_2 - V_1)$$

Constant T (PV is constant)

$$W_{12} = c \ln \frac{V_2}{V_1}$$

Adiabatic, reversible (Isentropic)

$$W_{12} = \frac{(P_2V_2 - P_1V_1)}{1 - \gamma}$$

Shaft Work, \dot{W}_{sh}

$$\dot{W}_{sh} = \text{Torque} \times \omega$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Constant Pressure/Volume

Constant Pressure $\frac{T_1}{T_2} = \frac{V_1}{V_2}$

Constant Volume $\frac{T_1}{T_2} = \frac{P_1}{P_2}$

Enthalpy

$$h = u + Pv \quad h = u + RT$$

Wet Vapour

Dryness Fraction, x

$$x = \frac{X - X_f}{X_g - X_f}$$

Specific Enthalpy, h

$$h = h_f + x h_{fg}$$

Specific Energy, u

$$u = u_f + x(u_g - u_f)$$

Specific Entropy, s

$$s = s_f + x(s_g - s_f)$$

Specific Volume, v

$$v = v_f + x(v_g - v_f)$$

The First Law

Always True

$$Q - W = (U_2 - U_1)$$

$$Q - W = m(u_2 - u_1)$$

For Cycles

$$\Sigma Q = \Sigma W$$

Constant V (no work)

$$Q = m(u_2 - u_1)$$

Perfect, Adiabatic & Reversible (Isentropic)

Pv^γ is constant

$$\frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^\gamma$$

$Tv^{\gamma-1}$ is constant

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$\frac{T}{P^{\frac{\gamma-1}{\gamma}}}$ is constant

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

Perfect Gasses & Temperature

Change in Energy, u $u_2 - u_1 = C_V(T_2 - T_1)$

Change in Enthalpy, h $h_2 - h_1 = C_P(T_2 - T_1)$

Perfect Gas $R = C_P - C_V$

Constants $\gamma = \frac{C_P}{C_V}$

$$C_P = \frac{R\gamma}{\gamma - 1}$$

Steady Flow Energy Equation

$$\dot{Q} - \dot{W}_{sh} = \sum_{out} \dot{m} \left[h + \frac{c^2}{2} + gZ \right] - \sum_{in} \dot{m} \left[h + \frac{c^2}{2} + gZ \right]$$

Mass Flow Rate, \dot{m} $\dot{m} = \rho cA$

$$\dot{m} = \frac{cA}{v}$$

Volume Flow Rate, \dot{V} $\dot{V} = \frac{\dot{m}}{\rho} = cA$

- ρ is the density
- c is the speed
- A is the cross-sectional area of the duct
- v is the specific volume

Clausius's Inequality

For a Reversible Cycle & Process:

$$\oint \frac{dQ}{T} = 0 \quad S_2 - S_1 = \int_1^2 \frac{dQ}{T}$$

For an Irreversible Cycle & Process:

$$\oint \frac{dQ}{T} < 0 \quad S_2 - S_1 > \int_1^2 \frac{dQ}{T}$$

Combining First & Second Laws

$$T ds = du + P dv$$

$$s_2 - s_1 = \int_1^2 \frac{1}{T} du + \int_1^2 \frac{P}{T} dv$$

$$T ds = dh - v dP$$

$$s_2 - s_1 = \int_1^2 \frac{1}{T} dh - \int_1^2 \frac{v}{T} dP$$

- These equations are valid for reversible and irreversible processes, as all quantities are properties

Perfect Gas Processes

$$s_2 - s_1 = C_V \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{v_2}{v_1} \right)$$

$$s_2 - s_1 = C_P \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

$$s_2 - s_1 = C_P \ln \left(\frac{v_2}{v_1} \right) + C_V \ln \left(\frac{P_2}{P_1} \right)$$

- Again, these apply to both reversible and irreversible processes

Actual Efficiency

General Efficiency, η

$$\eta = \frac{\text{output}}{\text{input}}$$

Thermal Efficiency η_{th}

$$\eta_{th} = \frac{W_{net}}{Q_H}$$

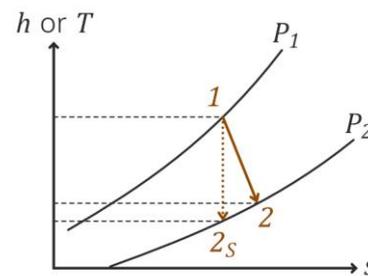
$$\eta_{th} = 1 - \frac{Q_C}{Q_H}$$

Reversible Efficiency

Carnot Efficiency

$$\eta_{th,rev} = 1 - \frac{T_C}{T_H}$$

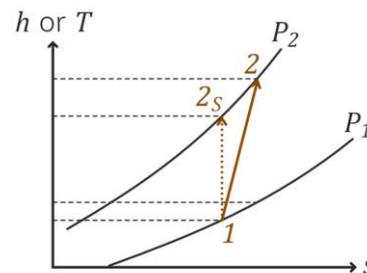
Isentropic Efficiency – Turbine/Engine



$$\eta_s = \frac{T_1 - T_2}{T_1 - T_{2s}}$$

$$\eta_s = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Isentropic Efficiency – Compressor/Pump



$$\eta_s = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$\eta_s = \frac{h_{2s} - h_1}{h_2 - h_1}$$