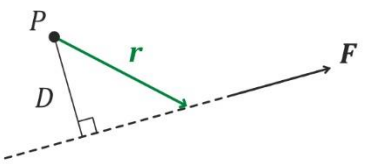
Mechanics Key Equations

Forces & Moments as Vectors

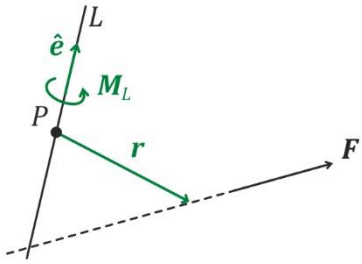


The magnitude of a moment about a point P in scalar form:

$$|\boldsymbol{M}_P| = |\boldsymbol{F}|D$$

In vector form, this is given as the cross product of the position and force vectors:

$$\boldsymbol{M}_P = \boldsymbol{r} \times \boldsymbol{F} \qquad |\boldsymbol{M}_P| = |\boldsymbol{r} \times \boldsymbol{F}|$$



Moments about a line:

$$\boldsymbol{M}_L = (\hat{\boldsymbol{e}} \,.\, \boldsymbol{r} \times \boldsymbol{F})\, \hat{\boldsymbol{e}}$$

$$\boldsymbol{M}_L = (\hat{\boldsymbol{e}} \,.\, \boldsymbol{M}_P)\, \hat{\boldsymbol{e}}$$

In both cases, the direction and sign of the moments are given by the right-hand rule.

Rectilinear Kinematics of Particles

The SUVAT equations for constant, linear acceleration:

$$v = u + at$$

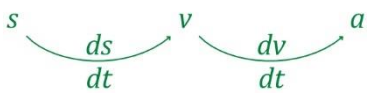
$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

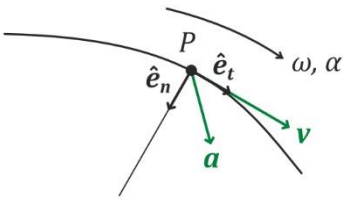
$$s = vt - \frac{1}{2}at^s$$

$$s = \frac{1}{2}(u + v)t$$

Integral relations:



Curvilinear Kinematics of Particles



Angular velocity & acceleration:

$$\omega = \dot{\theta} = \frac{d\theta}{dt}$$

$$\alpha = \ddot{\theta} = \frac{d^2\theta}{dt} = \frac{d\omega}{dt}$$

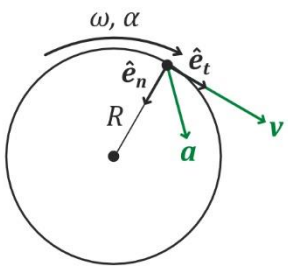
Unit conversion: $rad/s = RPM \times \frac{\pi}{30}$

Velocity is tangential:

$$v = r\omega \qquad \boldsymbol{v} = v\,\hat{\boldsymbol{e}}_{\boldsymbol{t}} = r\omega\,\hat{\boldsymbol{e}}_{\boldsymbol{t}}$$

Acceleration has both components:

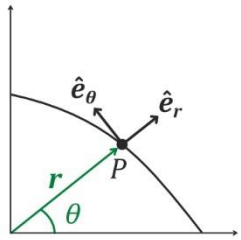
$$a = r\alpha \qquad \boldsymbol{a} = a\,\hat{\boldsymbol{e}}_{\boldsymbol{t}} + v\omega\,\hat{\boldsymbol{e}}_{\boldsymbol{n}} = a\,\hat{\boldsymbol{e}}_{\boldsymbol{t}} + \frac{v^2}{r}\,\hat{\boldsymbol{e}}_{\boldsymbol{n}}$$



For Circular motion:

$v = R\omega$ $\qquad$ $\boldsymbol{v} = R\omega\, \hat{\boldsymbol{e}}_{\boldsymbol{t}}$

$a = R\alpha$ $\qquad$ $\boldsymbol{a} = R\alpha\, \hat{\boldsymbol{e}}_{\boldsymbol{t}} + \frac{v^2}{R}\hat{\boldsymbol{e}}_n$



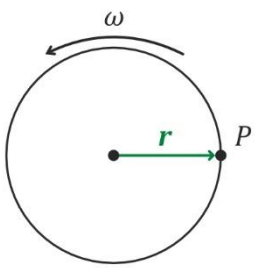
In Polar Coordinates:

$\boldsymbol{v} = \dot{r}\, \hat{\boldsymbol{e}}_r + r\dot{\theta}\, \hat{\boldsymbol{e}}_{\boldsymbol{\theta}}$

$\boldsymbol{a} = \left(\ddot{r} - r\dot{\theta}^2\right) \hat{\boldsymbol{e}}_{\boldsymbol{r}} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) \hat{\boldsymbol{e}}_{\boldsymbol{\theta}}$

Where $\boldsymbol{r} = r\hat{\boldsymbol{e}}_{\boldsymbol{r}}$

Kinematics of Rigid Bodies



Velocity of a point P on a rigid body:

$\boldsymbol{v}_P = \boldsymbol{\omega} \times \boldsymbol{r}$

Relative motion:

$\boldsymbol{v}_{A/B} = \boldsymbol{\omega} \times \boldsymbol{r}_{A/B}$

where $\boldsymbol{a}_{A/B} = \boldsymbol{a}_A - \boldsymbol{a}_B$

For a sliding contact at point A:

$\boldsymbol{v}_A = \boldsymbol{v}_B + \boldsymbol{\omega} \times \boldsymbol{r}_{A/B} + \boldsymbol{v}_r$

Where $\boldsymbol{v}_r$ is the velocity relative to the slot.

Acceleration of a point on a rigid body:

$\boldsymbol{a} = \boldsymbol{\alpha} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$

where $\boldsymbol{\alpha} \times \boldsymbol{r}$ is the tangential component and $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$ is the normal component.

For general plane motion, the normal component becomes:

$\boldsymbol{a} = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2 \boldsymbol{r}$

Relative motion:

$\boldsymbol{a}_{A/B} = \boldsymbol{\alpha} \times \boldsymbol{r}_{A/B} - \omega^2 \boldsymbol{r}_{A/B}$
where $\boldsymbol{a}_{A/B} = \boldsymbol{a}_A - \boldsymbol{a}_B$

For a sliding contact at point A:

$\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{\alpha} \times \boldsymbol{r}_{A/B} - \omega^2 \boldsymbol{r}_{A/B} + \boldsymbol{a}_r + 2\boldsymbol{\omega} \times \boldsymbol{v}_r$

Where $\boldsymbol{a}_r$ is the acceleration relative to the slot.

Friction

F is the frictional force, R the normal reaction with the surface:

$F = \mu R$

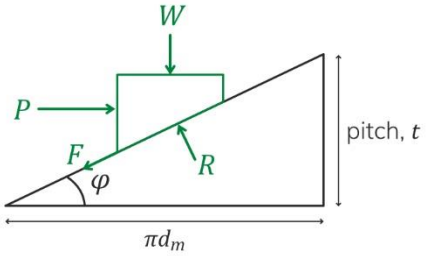
$\mu_s = \tan \alpha_{max}$

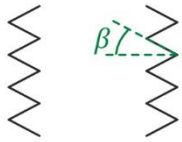
μ_s is the static coefficient of friction (~20% > μ_k, the kinetic coefficient of friction).

α_{max} is the friction angle – the maximum angle of inclination before the object slips:

Screw threads:

$$tan\phi = \frac{t}{\pi d_m}$$





Square Screw Threads:

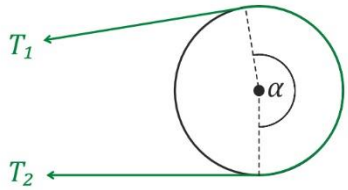
$$P = \left(\frac{tan\phi + \mu_s}{1 - \mu_s tan\phi}\right) \times W$$

For v-threads:

$$P = \left(\frac{tan\phi + \mu_s\, sec\beta}{1 - \mu_s\, sec\beta\, tan\phi}\right) \times W$$

Belts:

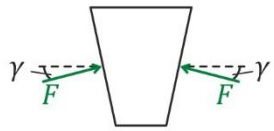
For square belts:



$$\frac{T_1}{T_2} = e^{\mu_s \alpha}$$

Where μ_s is the static coefficient of friction, and α the wrap angle.

For v-shaped belts:



$$\frac{T_1}{T_2} = e^{\frac{\mu_s \alpha}{sin\gamma}}$$

Where γ is the angle of the v-section.

Clutches

For uniform pressure:

$$F = \pi p(R_1^2 - R_2^2)$$

$$T = \frac{2}{3}\mu F\left(\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2}\right)$$

For uniform wear:

$$T = \mu F\left(\frac{R_1 + R_2}{2}\right)$$